Shri Vile Parle Kelavani Mandal's

[ITHIBAI COLLEGE OF ARTS, CHAUHAN INSTITUTE OF SCIENCE & AMRUTBE JIVANLAL COLLEGE OF COMMERCE AND ECONOMICS (AUTONOMOUS)

NAAC Reaccredited 'A' grade, CGPA: 3.57 (February 2016),
Granted under RUSA, FIST-DST & -Star College Scheme of DBT, Government of India,
Best College (2016-17), University of Mumbai

Affiliated to the

UNIVERSITY OF MUMBAI

Program: B.Sc. – Mathematics

S.Y. B.Sc.

Semester III & IV

Choice Based Credit System (CBCS) with effect from the Academic year 2022-23

A.C No: 13

Agenda No: 2.13

PREAMBLE

Mathematics has today become integral part of all industry domains as well as fields of academics and research. The industry requirements and technologies have been steadily and rapidly advancing in application of Mathematics. Organizations are increasingly opting for Machine learning techniques and Artificial intelligence which requires strong Mathematical background. The students are thinking beyond career in the industry and aiming for research opportunities. The B.Sc. Mathematics course structure therefore needed a fresh outlook and complete overhaul. A real genuine attempt has been made while designing the new syllabus for this 3- year graduate course. This syllabus prepares the students for a career in industry and also motivates them towards further studies and research opportunities.

The core philosophy of overall syllabus is to

(i) Form strong foundation of Mathematical science,
(ii) Introduce emerging trends to the students in gradual way
(iii) Groom the students for the challenges of industry.

The syllabus proposes to have nine core subjects of Mathematics. All core subjects are proposed to have theory as well as its application. The basic foundation of important skills required for Mathematical development is laid.

We sincerely believe that any student taking this course will get very strong foundation and exposure to basics, advanced and emerging trends of the subject. We hope that the students' community and teachers' fraternity will appreciate the treatment given to the courses in the syllabus.

We wholeheartedly thank all experts who shared their valuable feedbacks and suggestions in order to improvise the contents, we have sincerely attempted to incorporate each of them. We further thank Chairperson and members of Board of Studies for their confidence in us. Special thanks to Department of Mathematics and colleagues from various colleges, who volunteered or have indirectly helped designing certain specialized courses and the syllabus as a whole.

The curriculum retains the current workload of Mathematics Departments.

PROGRAMME SPECIFIC OUTCOMES (PSO'S)

On completion of the B.Sc in Mathematics, the learners should be enriched with knowledge and be able to-

PSO - 1 Acquire a strong foundation in various branches of mathematics to formulate real life problems into mathematical models.

PSO - 2 Develop problem solving skills, cultivating logical thinking, and face competitive examinations with confidence

PSO - 3 Enhance numerical ability and address problems in interdisciplinary areas which would help in project and field works.

PSO - 4 Apply the mathematical knowledge and skills to face competitive examination with confidence.

PSO – 5 Pursue higher studies which in turn will offer them job opportunities in government and public sector undertakings, banks, central government institutes etc.

PSO - 6 Develop entrepreneurial skills, become empowered and self dependent in society.

PSO – 7 Understand the professional, ethical, legal, security, social issues and responsibilities.

PSO – 8 Apply knowledge of principles, concepts and results in specific subject area to analyze their local and global impact.

PSO – 9 Communicate appropriately and effectively, in a scientific context using present technology and new findings.

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Semester –III

USMAMT 301 – Calculus – III

USMAMT 304 – Linear Algebra

USMAMT 303 – Higher Order Differential Equations

Semester -IV

USMAMT 401 – Calculus – IV

USMAMT 404 – Linear Algebra

USMAMT 403 – Graph Theory

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Evaluation Pattern

The performance of the learner will be evaluated in two components. The first component will be a Continuous Assessment with a weightage of 25% of total marks per course. The second component will be a Semester end Examination with a weightage of 75% of the total marks per course. The allocation of marks for the Continuous Assessment and End Semester Examinations is as shown below:

a) **Details of Continuous Assessment (CA)**

25% of the total marks per course:

Continuous Assessment	Details	Marks
Component 1 (CA-1)	Internal test	15 marks
Component 2 (CA-2)	Assignment	10 marks

b) **Details of End Semester Examination(ESE)**

75% of the total marks per course. Duration of examination will be two and half hours.

Question Number	Description	Marks	Total Marks
1	On Module I a) b) Attempt any two out of three each of 6 marks	a) 8 b) 12	20
2	On Module II a) b) Attempt any two out of three each of 6 marks	a) 8 b) 12	20
3	On Module III a) b) Attempt any two out of three each of 6 marks	a) 8 b) 12	20
4	On Module I, II,III Attempt any 3 out of 4 each of 5 marks	3 x 5=15	15
		Total Marks	**75**

AMishra

Mrs. Alka Mishra
Head, Department of Mathematics

MVaidya

Dr. Meenakshi Vaidya
Vice-Principal

KDesai

Dr. Krutika Desai
I/C Principal

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Program: B.Sc. (2022-23)				**Semester: III**	
Course: Calculus III				**Course Code: USMAMT301**	
Teaching Scheme				**Evaluation Scheme**	
Lecture Per week	**Practical (per week)**	**Tutorial (Hours per week)**	**Credit**	**Continuous Assessment (CA) (Marks - 25)**	**Semester End Examinations (SEE) (Marks- 75 in Question Paper)**
2	1 (2 hours/batch)	NIL	2	25	75

Learning Objectives:

1)To provide students specialist knowledge necessary for basic concepts in Calculus

2)Strive to enable students to learn basic concepts about functions of bounded variations and grasp basic concepts about Riemann Integration, sequences and series of functions

Course Outcomes:

After completion of the course, learners would be able to:

CO1: Explain, define and distinguish convergence and divergence of sequences and subsequences of real numbers

CO2: Develop the skill of analyzing sequences and series and evaluate convergence of series using different type of tests

CO3: Define and explain concept of limit points, closure, derived sets and continuity in R

CO4: Understand prove and apply properties of Riemann Integrabilty.

CO5: Understand and solve examples on Riemann integration.

CO6: Understand various types of improper integrals

CO7: Understand application of Riemann integrals

Outline of Syllabus: (per session plan)

Unit	**Description**	**No of lectures**
1	Real Numbers, Sequences and subsequences	10
2	Riemann Integration – I	10
3	Application of Riemann Integration and Improper integrals	10
	Total	**30**
PRACTICALS as per notifications of University		**15**

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Unit	Topic	No. of Lectures
Module 1	(i) Countable sets, Uncountable sets and examples The set of real numbers is uncountable. (ii) Neighborhood of a point, open sets in $\mathbb{R}$ and closed sets as complement of open sets. (iii)Limit point, examples, Closed interval contains all its limit points, A finite set has no limit points. Examples of infinite sets without any limit points (iv)Sequences, subsequences and properties (v) Limit inferior, Limit superior of a sequence (vi)Sequential characterization of a limit point of a set (vii) Bolzano-Weierstrass Theorem: Every Bounded sequence of real numbers has a convergent subsequence.	10
Module 2	(i) Approximation of area, Upper/lower Riemann sums and its properties, Upper/lower Riemann integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability (ii) If a < c < b then f $\in \mathcal{R}[a,b]$ iff f $\in \mathcal{R}[a,c]$ and f $\in \mathcal{R}[c,b]$. Further $\int_a^b f = \int_a^c f + \int_c^b f$ (iii)If f, g $\in \mathcal{R}[a,b]$ and $\alpha \in \mathbb{R}$ then (f + g) $\in \mathcal{R}[a,b]$ and α f $\in \mathcal{R}[a,b]$. Further $\int_a^b (f+g) = \int_a^b f + \int_a^b g$ and $\int_a^b \alpha f = \alpha \int_a^b f$ (iv)If f $\in \mathcal{R}[a,b]$ then |f| $\in \mathcal{R}[a,b]$ Further $\left\|\int_a^b f\right\| \leq \int_a^b \|f\|$ (v) If f ≥ 0 then $\int_a^b f \geq 0$ (vi)If f $\in \mathcal{C}[a,b]$ then f $\in \mathcal{R}[a,b]$ (vii) If f is bounded with finite number of discontinuities then f $\in \mathcal{R}[a,b]$ (viii) If f is monotonic then f $\in \mathcal{R}[a,b]$	10

Module 3	(i) Continuity of $F(x) = \int_a^x f(t)dt$ where $f \in R[a,b]$ (ii) Fundamental Theorems of Calculus (iii)Mean value theorems (iv)Integrations by parts, Leibnitz rule (v) Improper integrals – type 1 and type 2, Absolute convergence of improper integrals, Comparison tests (vi)β and Γ functions and their properties, relationship between β and Γ functions	10

PRACTICAL

One Practical per week per batch for the course of duration 2 hours (the batches to be formed as prescribed by the University).

Suggested Practicals:
1. Neighborhood of a point, Open/ closed sets
2. Limit points
3. Sequences and subsequences
4. Approximation of area
5. Upper and lower Riemann sum
6. Riemann integral properties and examples
7. Continuity of indefinite integral
8. Mean value theorems, Fundamental theorems of Calculus
9. Improper integrals
10. Beta, gamma functions

Essential Reference Books:

(1) Robert G. Bartle and Donald R. Sherbet : **Introduction to Real Analysis**

(Springer – Verlag)

(2) R. R. Goldberg : **Methods of Real Analysis**

(Oxford and IBH publication company, New Delhi)

(3) T. Apostol : **Calculus Vol. I, second edition** (John Wiley)

Supplementary Reference books:

(1) Thomas and Finney : Calculus and Analytic Geometry (Narosa Publications)

(2) T. Apostol : **Mathematical Analysis** (Narosa Publications)

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Program: B.Sc. (2022-23)					**Semester: III**
Course: Linear Algebra					**Course Code: USMAMT304**
Teaching Scheme				**Evaluation Scheme**	
Lectures (per week)	**Practical** (per week)	**Tutorial** (Hours per week)	**Credit**	**Continuous Assessment (CA) (Marks - 25)**	**Semester End Examinations (SEE) (Marks- 75 in Question Paper)**
2	1 (2 hours/batch)	NIL	2	25	75

Learning Objectives:

(1) Give the students a sufficient knowledge of fundamental principles by learning basic theorems and applications in Linear Algebra.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

Course Outcomes:

After completion of the course, learners would be able to:

CO1: Solve and analyze solution of systems of linear equations.

CO2: Perform elementary row operations on matrices, reduce matrices to echelon forms.

CO3: Solve systems of linear equations using augmented matrix/Gaussian Elimination

CO4: Understand basic concepts of vector spaces/subspaces and prove and apply related properties and theorems.

CO5: Understand/prove and apply basic concepts and theorems related to linear combinations of vectors, spanning and generating sets.

CO6: Understand/prove and apply basic concepts and theorems related to linearly independent/dependent sets of vectors, basis and dimension of vector spaces/subspaces.

CO7: Understand fundamental properties and theorems related to Linear transformations

CO8: Solve for Rank and Nullity of a linear transformation

Outline of Syllabus: (per session plan)

Module	**Description**	**No of lectures**
1	System of Linear Equations and Matrices	10
2	Vector spaces	10
3	Linear Transformations	10
	Total	30
PRACTICALS as per notifications of University		15

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Unit	Topic	No of lectures
Module 1	(i) Solutions of linear homogeneous and non-homogeneous systems of m equations in n unknowns and geometric interpretation for n = 2, 3. Scalar multiple and sums of solutions of a homogeneous system. Deduce that the homogeneous linear system of m equations in n unknowns has a non-trivial solution if $m < n$. (ii) Matrices with real entries, operations on matrices, transpose of a matrix, type of matrices. Inverse of a matrix. Elementary row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices. (iii) Row echelon and reduced row echelon matrix form, solve system of linear equations in matrix form and Gaussian elimination method.	10
Module 2	(i) Definition of a vector space V over $\mathbb{R}$, properties and examples. Subspaces and examples. Necessary and sufficient condition for a non-empty subset to be a subspace of a vector space.Intersection, union, sum and direct sum of subspaces. (ii) Linear combinations of vectors, linear span of a non-empty set S, generating set, L(S) is a subspace. Linearly independent/dependent vectors. (iii) Basis and dimension of a finitely generated vector space and subspaces. Any two basis sets of a vector space have same number of vectors. Basis as minimal generating set/maximal linearly independent set. Basis of a subspace can be extended to basis of the vector space. If U and W are subspaces of a vector space then $dim(U + W) = dimU + dimW - dim(U \cap W)$.	10
Module 3	(i) Linear transformations from V to W where V, W are finite dimensional real vector spaces. Properties and examples of linear transformations. A linear transformation is completely determined by its values on a basis. (ii) Sums, scalar multiples and composition of linear transformations. Kernel and image of a linear transformation, Rank-Nullity Theorem. (iii) Invertible linear transformations, linear isomorphisms and any two vector spaces having same dimension are isomorphic to each other.	10

PRACTICAL

One Practical per week per batch for the course of duration 2 hours (the batches to be formed as prescribed by the University).

Suggested Practicals:
1. Solution of linear systems
2. Matrices
3. Row echelon forms and Gaussian elimination
4. Vector spaces
5. Subspaces
6. Linear combinations and span of a set
7. Linear independence, basis and dimension
8. Linear transformations
9. Rank-Nullity theorem
10. Linear isomorphisms

Essential Reference Book:

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. K.Hoffmann and R. Kunze Linear Algebra, Tata MacGraw Hill, New Delhi

Supplementary Reference Books:

1. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India Pvt. Ltd,
2. Gilbert Strang, Linear Algebra and it's Applications, International Student Edition.
3. L. Smith, Linear Algebra, Springer Verlang.
4. A. Ramchandran Rao, P. Bhimashankaran; Linear Algebra Tata Mac Graw Hill.

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Program: B.Sc . (2022-23)				Semester: III	
Course: Higher Order Differential Equations				Course Code: USMAMT303	
Teaching Scheme				Evaluation Scheme	
Lectures (per week)	**Practical** (per week)	**Tutorial** (Hours per week)	**Credit**	**Continuous Assessment (CA)** (Marks - 25)	**Semester End Examinations (SEE)** (Marks- 75 in Question Paper)
2	1 (2 hours/batch)	NIL	2	25	75

Learning Objectives:

(1) Give the students a sufficient knowledge of different methods and concepts.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

Course Outcomes:

After completion of the course, learners would be able to:

CO1: Understand Different Basic Concepts, definitions & types related to Second Order Differential equations

CO2: Understand Solution space and General solution of Homogenous Differential equations

CO3: Understand Terms Wronskian, linearly dependent, linearly Independent and different theorems related to solution of Differential equations

CO4: Apply Methods Undetermined Coefficient and variation of parameters of Non Homogenous Differential equations

CO5: Understand Definition and Concept of Inverse Operator

CO6: Understand Operator Methods for Standard functions

CO7: Apply and solve Euler Cauchy & Legendre equation

CO8: Understand Different Basic Concepts, definitions & methods to solve System of Differential equations

CO9: Understand Sturm Separation and Comparison theorem and able to apply in different mathematical problems

Outline of Syllabus: (per session plan)

Module	Description	No of lectures
1	Second Order Linear Differential Equations with constant coefficients	10
2	Operators and Standard Linear differential equations with variable coefficients	10
3	System of Differential Equations	10
	Total	**30**
PRACTICALS		**15**

Unit	Topic	No. of Lectures
Module 1	(i) The general second order differential equation. Existence and Uniqueness Theorem for the solution of Second order initial value problem (Statement Only) (ii) Homogeneous and non – nonhomogeneous higher order differential Equations. Solution space of homogeneous equations as a vector space Wronskian and linear independence of the solutions. The general solution of a homogeneous differential equation. The use of known solution to find a general solution of homogeneous equation. The general solution of a Non-homogeneous second order differential equation. (iii) The homogeneous equation with constant coefficients, auxiliary equation, the general solution corresponding to real, distinct, complex and repeating roots. (iv) Non-homogeneous equation: The method of undetermined coefficients, the Method of variation of parameters.	10
Module 2	(i) Concept of Inverse operator. (ii) Method of operators for standard functions. (iii) (i) $\frac{1}{D-a}e^{ax} = xe^{ax}$, (ii) $\frac{1}{(D-a)^n}e^{ax} = \frac{x^n}{n!}e^{ax}$ (iii) $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$, if $f(a) \neq 0$ (iv) $\frac{1}{f(D)}e^{ax} = \frac{x^r}{r!\phi(a)}e^{ax}$, if $f(D) \neq (D-a)^r\phi(D)$ where $\phi(a) \neq 0$ (iv) (i) $\frac{1}{f(D^2)}\sin ax = \frac{1}{f(-a^2)}\sin ax$ and $\frac{1}{f(D^2)}\cos ax = \frac{1}{f(-a^2)}\cos ax$, if $f(-a^2) \neq 0$ (v) Inverse operator for polynomials (vi) $\frac{1}{f(D)}(e^{ax}V) = e^{ax}\frac{1}{f(a+D)}V$ where V is a function of x. (vii) Euler- Cauchy equation, Legendre's equation.	10

Module 3	(i) Existence and uniqueness theorem (Statement only) (ii) Study of homogeneous system of ODEs in two variables. Let $a_1(t)$, $a_2(t)$, $b_1(t)$, $b_2(t)$ be continuous real valued functions defined on [a, b]. Fix $t_0 \in [a, b]$. Then there exists a unique solution $x = x(t)$, $y = y(t)$ valid throughout [a, b] of the following system : $\frac{dx}{dt} = a_1(t)x + b_1(t)y$ $\frac{dy}{dt} = a_2(t)x + b_2(t)y$ Satisfying the initial condition: $x(t_0) = x_0$ and $y(t_0) = y_0$. (iii)The Wronskian W(t) of two solutions of homogeneous linear system of ODEs in two variables, result : W(t) is identically zero or nowhere zero on [a, b]. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables. (iv)Explicit solutions of a homogeneous linear system with constant coefficients in two variables. (v) Sturm's Separation Theorem, Sturm's comparison Theorem and applications.	10

Essential Reference Books:

(1) G. F. Simmons: Differential Equations with applications and historical notes (McGraw Hill)
(2) Erwin Kreyszig: Advanced Engineering Mathematics

Supplementary Reference Books

E. A. Codington: An introduction to ordinary differential equations (Dover Books)

Suggested Practicals

1. Homogeneous differential equations
2. Wronskian, Linearly dependent and independent solutions
3. General solution of non-homogeneous differential equations by Undetermined Coefficients
4. General solution of non-homogeneous differential equations by Variation of Parameters method
5. Operator method for exponential and sine/cosine functions
6. Polynomials and functions of the type $e^{ax}V$ where V is a function of x.
7. Euler- Cauchy and Legendre's equation.
8. System of differential equations
9. System of differential equations general method
10. Sturm's Separation Theorem , Sturm's comparison Theorem

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Program: B.Sc . (2022-23)					
Course: Calculus IV				Semester: IV	
				Course Code: USMAMT401	
Teaching Scheme				Evaluation Scheme	
Lectures (Per week)	Practical (per week)	Tutorial (Hours per week)	Credit	Continuous Assessment (CA) (Marks - 25)	Semester End Examinations (SEE) (Marks- 75 in Question Paper)
2	1 (2 hours/batch)	NIL	2	25	75

Learning Objectives:

(1) Give the students a sufficient knowledge of continuity and differentiability of a scalar and vector fields

(2) To study application of differential calculus in n–dimensional space.

Course Outcomes:

After completion of the course, learners would be able to:

CO1: Define scalar fields and its limit, continuity and derivative at a point
CO2: Define directional derivative, gradient and establish relation between them
CO3: Illustrate Geometric meaning of various types of derivatives
CO4: Define partial derivative and mixed partial derivative
CO5: Solve problems to find extreme values in 2 dimensions
CO6: Define vector fields and its derivative at a point
CO7: Apply chain rule for both scalar and vector fields

Outline of Syllabus: (per session plan)

Module	Description	No of lectures
1	Functions of Several Variables	10
2	Differentiation of scalar fields and vector fields	10
3	Differentiation of vector fields and Application of differential calculus	10
	Total	**30**
PRACTICALS as per University Rules		**15**

Unit	Topic	No. of Lectures
Module 1	(i) Review of functions from $\mathbb{R}^n$ to $\mathbb{R}$ (scalar fields), Iterated limits. (ii) Limits and continuity of functions from $\mathbb{R}^n$ to R (iii)Basic results on limits and continuity of sum, difference, scalar multiples of vector fields. (iv)Continuity and components of vector fields.	10
Module 2	(i) Derivative of a scalar field with respect to a vector. (ii) Direction derivatives and partial derivatives of scalar fields. (iii)Mean value theorem for derivatives of scalar fields. (iv)Differentiability of a scalar field at a point (in terms of linear transformation). (v) Total derivative. Uniqueness of total derivative of a differentiable function at a point. Examples. Differentiability at a point implies continuity, and existence of direction derivative at the point. The existence of continuous partial derivatives in a neighborhood of a point implies differentiability at the point. (vi)Gradient of a scalar field. Geometric properties of gradient, level sets and tangent planes. Chain rule for scalar fields. (vii) Higher order partial derivatives, mixed partial derivatives. Sufficient condition for equality of mixed partial derivatives. (viii) Second order Taylor formula for scalar fields.	10
Module 3	(i) Definition of differentiability of a vector field at a point. Differentiability of a vector field at a point implies continuity. (ii) The chain rule for derivative of vector fields (statement only). (iii)Maxima, minima, saddle point, Second derivative test for extrema of function of two variables (iv)Method of Lagrange multipliers.	10

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

PRACTICAL

One Practical per week per batch for the course of duration 2 hours (the batches to be formed as prescribed by the University).

Suggested Practicals

1. Functions from $\mathbb{R}^n$ to $\mathbb{R}$
2. Limits and continuity of scalar functions
3. Continuity and components of vector functions
4. Derivative of a scalar field w.r.t. a vector
5. Directional and partial derivatives of a scalar field
6. Gradient and chain rule of a scalar field
7. Higher order partial derivatives
8. Mixed order partial derivatives
9. Maxima, minima, saddle point, second derivative test
10. Method of Lagrange multipliers

Essential Reference Books:

1. Robert G. Bartle and Donald R. Sherbet : **Introduction to Real Analysis** (Springer – Verlag)
2. R. R. Goldberg : **Methods of Real Analysis** (Oxford and IBH publication company, New Delhi)
3. T. Apostol : **Calculus Vol. I, second edition** (John Wiley)

Supplementary Reference books:

1. Thomas and Finney : Calculus and Analytic Geometry (Narosa Publications)
2. T. Apostol : **Mathematical Analysis** (Narosa Publications)
3. Calculus. J. Stewart. Brooke/Cole Publishing Co.

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Program: B.Sc . (2022-23)					**Semester: IV**
Course: Linear Algebra					**Course Code: USMAMT404**
Teaching Scheme				**Evaluation Scheme**	
Lecture (per week)	**Practical** (per week)	**Tutorial** (Hours per week)	**Credit**	**Continuous Assessment (CA)** (Marks - 25)	**Semester End Examinations (SEE)** (Marks- 75 in Question Paper)
2	1 (2 hours/batch)	NIL	2	25	75

Learning Objectives:

(1) Give the students a sufficient knowledge of fundamental principles by learning basic theorems and applications in Linear Algebra.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

Course Outcomes:

After completion of the course, learners would be able to:

CO1: Understand basic concepts and apply theorems on matrix associated with a linear transformation.

CO2: Find rank of a matrix and apply to determine existence of a solution and solution space of associated linear system.

CO3: Find determinant of a square matrix and apply to check linear independence of vectors, find inverse of a matrix and solve systems of linear equations.

CO4: Understand basic concepts and theorems related to eigenvalues, eigenvectors and diagonalizable linear transformations and matrices.

CO5: Understand basic concepts and theorems related to inner product on a vector space, norm of a vector, orthogonal and orthonormal sets and bases and apply to solve problems.

CO6: Apply Gram-Schmidt orthogonalization process

Outline of Syllabus: (per session plan)

Module	**Description**	**No of Hours**
1	Matrix of a Linear Transformation and Determinant	10
2	Eigenvalues and Diagonalization	10
3	Inner Product Spaces	10
	Total	**30**
PRACTICALS as per University Rules		**15**

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Unit	Topic	No of lectures
Module 1	(i) Co-ordinate vectors w.r.t. a given ordered basis of a finite dimensional real vector space V. Representation of a linear transformation from U to V (where U and V are finite dimensional real vector spaces) by matrices w.r.t. the given ordered bases of U and V. The relation between the matrices of a linear transformation from U to U with respect to di□erent bases of U. Matrix of sums, scalar multiple, composite and inverse of linear transformations. (ii) Rank of a $m \times n$ matrix with real entries. Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_A : \mathbb{R}^n \to \mathbb{R}^m$ $(LA(X) = AX)$. Dimension of solution space of the system of linear equations $AX = 0$ equals $n - rank\ A$. Existence of solution of the non-homogeneous system $AX = B$ when $rank(A) = rank\ (A^*)$. The general solution $AX = B$ in terms of a particular solution of the system and solutions of the associated homogeneous system. (iii)Determinant of a square matrix using Laplace expansion. Properties of determinants. Linear independence/ dependence of vectors in $\mathbb{R}^n$ using determinants. Adjoint of an $n \times n$ matrix A, $Adj(A) = det(A)I_n$, Inverse of A. Existence and uniqueness of solution of the system $AX = B$, where A is an $n \times n$ matrix with $\det(A) \neq 0$, Cramer's rule.	10
Module 2	(i) Eigenvalues and eigenvectors of a linear transformation $T: V \to V$ where V is a finite dimensional real vector space and examples. Eigenvalues and eigenvectors of $n \times n$ real matrices, linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation/matrix. (ii) Characteristic polynomial of an $n \times n$ real matrix. Result: A real number is an eigenvalues of an $n \times n$ matrix A if and only if λ is a root of the characteristic polynomial of A. Cayley-Hamilton Theorem (statement only), Similar matrices, Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices. (iii)Diagonalizability of a linear transformation of a finite dimensional real vector space to itself and of a $n \times n$ real matrix. Characterization of diagonalization of a L.T. on $\mathbb{R}^n$ using a basis of eigenvectors Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix and of a linear transformation. Examples of non-diagonalizable matri A $n \times n$ real matrix A is diagonalizable if and only if the	10

	algebraic and geometric multiplicities of eigenvalues of A coincide.	
Module 3	(i) Dot product in $\mathbb{R}^n$, definition of general inner product on a vector space over $\mathbb{R}$. Examples of inner product including the inner product $< f, g > = \int_{\pi}^{\pi} f(t)g(t)dt$ on $C[-\pi, \pi]$, (ii) Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle in- equality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^2$, Projections, the projection being the closest approximation. (iii) Orthogonal and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram- Schmidt orthogonalization process, Simple examples in $\mathbb{R}^3$, $\mathbb{R}^4$.	10

PRACTICAL

One Practical per week per batch for the course of duration 2 hours (the batches to be formed as prescribed by the University).

Suggested Practicals:
1. Matrix associated with a linear transformation
2. Rank of a matrix A and Rank of the linear transformation L_A
3. Determinants
4. Adjoint of a matrix, Inverse of a matrix using determinant, Cramer's rule
5. Eigenvalues and eigenvectors
6. Characteristic polynomials
7. Diagonalization
8. Inner product spaces
9. Orthogonality and Projection
10. Orthonormal sets and Gram-Schmidt process

Essential Reference Book:

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. K.Hoffmann and R. Kunze Linear Algebra, Tata MacGraw Hill, New Delhi

Supplementary Reference Book:

1. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India Pvt. Ltd,
2. Gilbert Strang , Linear Algebra and it's Applications, International Student Edition.
3. L. Smith , Linear Algebra, Springer Verlang.
4. A. Ramchandran Rao, P. Bhimashankaran; Linear Algebra Tata Mac Graw Hill.

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Program: B.Sc . (2022-23)				**Semester: III**	
Course: Graph Theory				**Course Code: USMAMT403**	
Teaching Scheme				**Evaluation Scheme**	
Lecture (per week)	**Practical** (per week)	**Tutorial** (Hours per week)	**Credit**	**Continuous Assessment (CA)** (Marks - 25)	**Semester End Examinations (SEE)** (Marks- 75 in Question Paper)
2	1 (2 hours/batch)	NIL	2	25	75

Learning Objectives:

(1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of immense power of mathematical ideas.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

Course Outcomes:

After completion of the course, learners would be able to:

CO1: Understand Basic definitions and results of Graph Theory, able to apply and Solve Problems in Basic Graph Theory

CO2: Understand Isomorphic Graphs, Adjacency & Incidence Matrix

CO3: Apply Havel Hakimi and Dijkstra's Algorithm

CO4: Understand Basic definitions and results of Trees

CO5: Understand Minimal Spanning Tree, Prefix Code and Huffman Coding

CO6: Apply BFS, DFS, Prims and Kruskals Algorithm

CO7: Determine Whether Graphs are Euler and/or Hamilton

CO8: Understand different theorems for finding Euler path, Euler circuit, Hamilton Path and Hamilton circuit

Outline of Syllabus: (per session plan)

Module	**Description**	**No of lectures**
1	Basics of Graphs	10
2	Eulerian and Hamiltonian graphs	10
3	Trees	10
	Total	**30**
PRACTICALS		**15**

Unit	Topic	No. of lectures
Module 1	(i) Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, (ii) Isomorphism between the graphs and consequences of isomorphism between the graphs, Self-complementary graphs, Connected graphs, Connected components. (iii) Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problem, Dijkstra's algorithm.	10
Module 2	(i) Eulerian graph and its characterization- Fleury's Algorithm-(Chinese postman problem), (ii) Hamiltonian graph, Necessary condition for Hamiltonian graphs using G-S where S is a proper subset of V(G) (Only Statement), Sufficient condition for Hamiltonian graphs-Ore's theorem (Only Statement). Dirac's theorem (Only Statement), Hamiltonian closure of a graph, (iii) Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.	10
Module 3	(i) Cut edges and cut vertices and relevant results, Characterization of cut edge, (ii) Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of , Algorithms for spanning tree-BFS and DFS, Binary and m-ary tree, Prefix codes and Huffman coding, (iii) Weighted graphs and minimal spanning trees - Kruskal's algorithm for minimal spanning trees.	10

SVKM's Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce & Economics (AUTONOMOUS)

Essential Reference Book:

Kenneth H. Rosen, Discrete Mathematics and its Applications

Supplementary Reference Books:

1. Bondy and Murty Grapgh, Theory with Applications.
2. Balkrishnan and Ranganathan, Graph theory and applications.

PRACTICAL

Suggested Practicals

1. Problems in Basic Graph Theory & Havel Hakimi Algorithm
2. Isomorphic Graphs, Bipartite and complement
3. Shortest path and Dijkstra's Algorithm
4. Strongly and Weekly Connected Components
5. Adjacency & Incidence Matrix and number of Paths
6. Euler and/or Hamilton Graphs
7. Understand different theorems for finding Euler path, Euler circuit, Hamilton Path and Hamilton circuit
8. Basic Examples on Trees
9. BFS, DFS, Prefix Code and Huffman Coding
10. Minimal Spanning Tree Prims and Kruskals Algorithm